

Application and Performance Analysis of an S-curve Trajectory Planning Algorithm Based on Sine Functions in High-Speed Transfer Equipment

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Abstract: To address the high demands for trajectory smoothness and transfer efficiency during the motion of high-speed, high-precision transfer equipment for flexible materials, traditional trajectory planning methods often face issues such as abrupt acceleration changes, significant impact, or computational complexity. Based on a project technical report, this paper proposes an S-curve trajectory planning algorithm based on sine functions. By designing a continuously smooth sine acceleration curve, the algorithm divides the motion process into five stages: acceleration-jerk, acceleration-deceleration, constant velocity, deceleration-jerk, and deceleration-deceleration, and establishes an accurate mathematical model. Through MATLAB simulation, this algorithm is compared and analyzed against cubic polynomial, quintic polynomial, and trapezoidal velocity planning algorithms. The results show that this algorithm achieves continuous and smooth transitions in acceleration while ensuring high transfer efficiency, effectively suppressing impact and vibration. It offers the optimal comprehensive performance and is suitable for high-speed transfer scenarios involving flexible materials.

Introduction

With the development of intelligent manufacturing, the demand for automated cutting and transfer of flexible materials (such as fabrics, films, composite materials, etc.) is growing rapidly. In smart unmanned factories for flexible material cutting, high-speed, precise transfer equipment must complete the g ripping, transferring, and placing of cut pieces. However, impact and vibration generated during the equipment's start-stop and variable-speed processes can cause damage to soft, easily deformable materials, affecting product quality. Therefore, designing a motion trajectory planning algorithm that combines high transfer efficiency and high smoothness is key to improving equipment performance.

Currently, common trajectory planning methods include cubic polynomial, quintic polynomial,

and trapezoidal velocity planning. These methods each have limitations when applied to the high-speed, high-precision transfer of flexible materials. To resolve existing problems, this project proposed an innovative S-curve trajectory planning algorithm based on sine functions in its report, aiming to optimize the acceleration curve and realize smooth, rapid motion for the equipment.

1. Analysis of Limitations in Common Trajectory Planning Methods

Before designing the algorithm, this report conducted a systematic comparative analysis of three common methods:

- Cubic Polynomial Planning: Based on a cubic function of time for displacement, it solves the trajectory by satisfying initial and terminal displacement and velocity constraints. Its advantage lies in simple calculation and good real-time performance, achieving continuous velocity. However, its disadvantage is that acceleration is discontinuous, easily producing impact and vibration. It is not suitable for transferring flexible materials that require high smoothness and is only applicable to low-speed, light-load scenarios^[1].

- Quintic Polynomial Planning: Based on a quintic function of time for displacement, it solves the trajectory by satisfying initial and terminal displacement, velocity, and acceleration constraints. Its advantage is that acceleration is continuous, impact is smaller, and motion smoothness is superior to cubic polynomials. However, the computational complexity is high, real-time performance is poor, the coefficient solving process is cumbersome, and maximum acceleration is difficult to control. It is suitable for medium-speed, medium-load scenarios.

- Trapezoidal Velocity Planning: The motion process is divided into acceleration, constant velocity, and deceleration stages, with a trapezoidal velocity curve. Its advantages include flexible velocity adjustment (the constant velocity can be adjusted based on transfer efficiency needs) and moderate computational difficulty. However, abrupt changes in acceleration exist between the acceleration and constant velocity segments, and between the constant velocity and deceleration segments, easily producing impact and significant vibration. It is suitable for high-speed, heavy-load scenarios where moderate smoothness is acceptable.

Through comparative analysis, it is evident that the three common methods above cannot fully satisfy this project's requirements for high smoothness and high real-time performance in flexible material transfer. Based on this, this project proposed an S-curve trajectory planning algorithm based on sine functions.

2. Design of the S-curve Trajectory Planning Algorithm Based on Sine Functions

The core of S-curve trajectory planning is to divide the motion process into acceleration, constant velocity, and deceleration stages, and optimize the acceleration curve to ensure a smooth transition of acceleration, avoid abrupt changes, and thereby reduce impact and vibration. The S-curve trajectory planning based on sine functions proposed in this project uses the sine function and its derivative as the core of the acceleration curve, making the acceleration in the acceleration and deceleration stages change along a sine curve. This achieves a continuous and smooth transition of acceleration while ensuring stable velocity in the constant velocity stage, balancing both transfer smoothness and efficiency.

2.1 Division of Motion Stages

The motion process of the S-curve trajectory is divided into five stages: acceleration-jerk, acceleration-deceleration, constant velocity, deceleration-jerk, and deceleration-deceleration^[2].

- Acceleration-Jerk: Acceleration starts from 0 and gradually increases to the maximum

acceleration a_{max} according to the sine function law.

- Acceleration-Deceleration: Acceleration gradually decreases to 0 according to the sine function law, and the velocity reaches the maximum velocity v_{max} .

- Constant Velocity: Acceleration remains at 0, and velocity is maintained at v_{max} to achieve rapid transfer.

- Deceleration-Jerk: Acceleration increases in the reverse direction to $-a_{max}$ according to the sine function law, and velocity begins to decrease.

- Deceleration-Deceleration: Acceleration decreases in the reverse direction to 0 according to the sine function law, and velocity drops to 0, reaching the target position.

2.2 Acceleration Function Design

Let the total motion time be T , the total acceleration time be T_a (where the acceleration-jerk time T_{a1} and the acceleration-deceleration time T_{a2} are equal, i.e., $T_{a1}=T_{a2}=T_d/2$), the total deceleration time be T_d (where the deceleration-jerk time T_{d1} and the deceleration-deceleration time T_{d2} are equal, i.e., $T_{d1}=T_{d2}=T_d/2$), the constant velocity time be $T_v=T-T_a-T_d$, and the maximum acceleration be a_{max} . The acceleration expressions for each stage are as follows (see Figure 1):

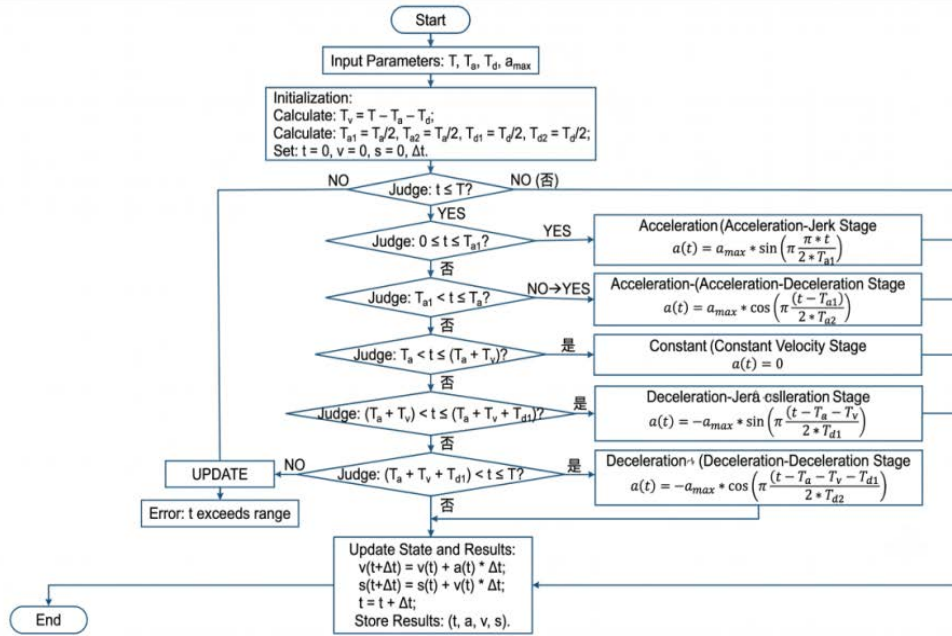


Figure 1: the procedure of acceleration function

- Acceleration-Jerk ($0 \leq t \leq T_{a1}$): $a(t) = a_{max} * \sin(\pi * t / (2 * T_{a1}))$. In this stage, acceleration starts from 0 and increases according to the sine function. When $t = T_{a1}$, acceleration reaches the maximum value a_{max} , achieving a smooth start of acceleration.

- Acceleration-Deceleration ($T_{a1} < t \leq T_a$): $a(t) = a_{max} * \cos(\pi * (t - T_{a1}) / (2 * T_{a2}))$. In this stage, acceleration decreases according to the cosine function. When $t = T_a$, acceleration drops to 0, and velocity reaches the maximum velocity v_{max} , achieving a smooth transition from the acceleration stage to the constant velocity stage.

- Constant Velocity ($T_a < t \leq T_a + T_v$): $a(t) = 0$. In this stage, acceleration remains at 0, and velocity is maintained at v_{max} , maximizing transfer efficiency.

- Deceleration-Jerk ($T_a + T_v < t \leq T_a + T_v + T_{d1}$): $a(t) = -a_{max} * \sin(\pi * (t - T_a - T_v) / (2 * T_{d1}))$. In this stage, acceleration increases in the reverse direction, decreasing from 0 to $-a_{max}$ according to the sine

function, and velocity begins to decrease smoothly.

•Deceleration-Deceleration ($T_a + T_v + T_{d1} < t \leq T$): $a(t) = a_{max} * \cos(\pi * (t - T_a - T_v - T_{d1}) / (2 * T_{d2}))$. In this stage, acceleration decreases in the reverse direction, rising from $-a_{max}$ to 0 according to the cosine function. When $t = T$, velocity drops to 0, reaching the target position and achieving a smooth transition from the deceleration stage to the stopped state^[3].

2.3 Derivation of Kinematic Equations

Based on the acceleration expressions above, velocity expressions for each stage can be obtained through integration. Integrating the velocity expressions further provides displacement expressions for each stage.

•Acceleration-Jerk Velocity: $v(t) = (2 * a_{max} * T_{a1} / \pi) * (1 - \cos(\pi * t / (2 * T_{a1})))$.

•Acceleration-Deceleration Velocity: $v(t) = v(T_{a1}) + ((2 * a_{max} * T_{a2} / \pi) * \sin(\pi * (t - T_{a1}) / (2 * T_{a2})))$.

When $t = T_a$, velocity reaches the maximum value $v_{max} = 2 * a_{max} * T_a / \pi$.

•Constant Velocity Stage Velocity: $v(t) = v_{max}$

•Deceleration-Jerk Velocity: $v(t) = V_{max} - (2 * a_{max} * T_{d1} / \pi) * (1 - \cos(\pi * (t - T_a - T_v) / (2 * T_{d1})))$

•Deceleration-Deceleration

Velocity:

$v(t) = v(T_a + T_v + T_{d1}) + (2 * a_{max} * T_{d2} / \pi) * \cos(\pi * (t - T_a - T_v - T_{d1}) / (2 * T_{d2}))$. When $t = T$, velocity $v(T) = 0$, satisfying terminal velocity constraints.

2.4 Determination of Core Parameters

The core parameters of the sine-function-based S-curve trajectory planning algorithm include maximum acceleration a_{max} , maximum velocity v_{max} , acceleration time T_a , deceleration time T_d , and constant velocity time T_v . The determination of these parameters must combine the mechanical performance of the transfer equipment, the characteristics of the flexible materials, and the requirements for transfer efficiency^[4].

•Maximum Acceleration (a_{max}): Determined based on the motor power of the transfer equipment, the load-bearing capacity of the transmission mechanism, and the impact resistance of the flexible materials, avoiding excessive acceleration that could cause equipment vibration or material damage. Through experimental testing, this project determined $a_{max} = 2m/s^2$.

•Maximum Velocity (v_{max}): Combined with transfer efficiency requirements and the maximum operating speed of the equipment, the maximum velocity is increased as much as possible while ensuring smoothness^[4]. This project determined $v_{max} = 1m/s$. Based on $v_{max} = 2 * a_{max} * T_a / \pi$, the acceleration time $T_a = \pi * v_{max} / (2 * a_{max}) \approx 0.785s$ can be calculated, thus $T_{a1} = T_{a2} = 0.3925s$.

•Deceleration Time (T_d): To ensure the symmetry and smoothness of the motion, $T_d = T_a \approx 0.785s$ is taken, i.e., $T_{d1} = T_{d2} = 0.3925s$.

•Constant Velocity Time (T_v): Calculated based on the distance S between the initial point and the target point. First, calculate the total displacement $S_a + S_d$ of the acceleration and deceleration stages, then calculate the constant velocity stage displacement $S_v = S - (S_a + S_d)$, and finally obtain the constant velocity time through $T_v = S_v / v_{max}$.

3. Simulation Verification and Performance Analysis

To verify the feasibility and superiority of the sine-function-based S-curve trajectory planning algorithm, this project conducted MATLAB simulation experiments, comparing the algorithm with cubic polynomial, quintic polynomial, and trapezoidal velocity planning algorithms, analyzing them from the three dimensions of trajectory smoothness, impact degree, and transfer efficiency.

The simulation experiment was based on the transfer path designed by this project, with an initial

point $P_0(0, 0, 0)$, a target point $P_3(1000\text{mm}, 500\text{mm}, 300\text{mm})$, and process path points $P_1(300\text{mm}, 150\text{mm}, 100\text{mm})$ and $P_2(700\text{mm}, 350\text{mm}, 200\text{mm})$, with a total transfer distance of $s \approx 1118\text{mm}$. The simulation parameters for each algorithm were set uniformly as: maximum velocity $v_{\max} = 1\text{m/s}$ and maximum acceleration $a_{\max} = 2\text{m/s}^2$, ensuring fairness in the comparison.

3.1 Analysis of Trajectory Smoothness

By comparing the acceleration and velocity curves of each algorithm, the smoothness of the trajectories was evaluated. The acceleration curve of the sine-function-based S-curve trajectory is continuous and smooth, with no abrupt changes, and the velocity curve transitions naturally. The acceleration curve of the cubic polynomial has abrupt changes; while the velocity curve is continuous, its smoothness is poor. The acceleration curve of the quintic polynomial is continuous, but the velocity curve does not transition naturally enough. The acceleration curve of the trapezoidal velocity planning has obvious abrupt changes, and the velocity curve has inflection points at the stage connections^[5].

3.2 Analysis of Impact Degree

The impact degree is measured by the rate of change of acceleration (jerk); the smaller the jerk, the smaller the impact. The sine-function-based S-curve trajectory has the smallest maximum jerk. The cubic polynomial has an infinite jerk (acceleration abrupt change). The quintic polynomial has a maximum jerk greater than this algorithm. The trapezoidal velocity planning has an infinite jerk (acceleration abrupt change). It is evident that the sine-function-based S-curve trajectory has the smallest impact, effectively reducing equipment vibration and mechanical wear.

3.3 Analysis of Transfer Efficiency

Transfer efficiency is measured by the total transfer time; the shorter the total time, the higher the efficiency. Through simulation calculations, the total transfer time of the four algorithms is compared as follows:

- Sine-function-based S-curve: 1.97s.
- Cubic Polynomial: $\sim 2.05\text{s}$.
- Quintic Polynomial: $\sim 2.12\text{s}$.
- Trapezoidal Velocity Planning: $\sim 1.95\text{s}$.

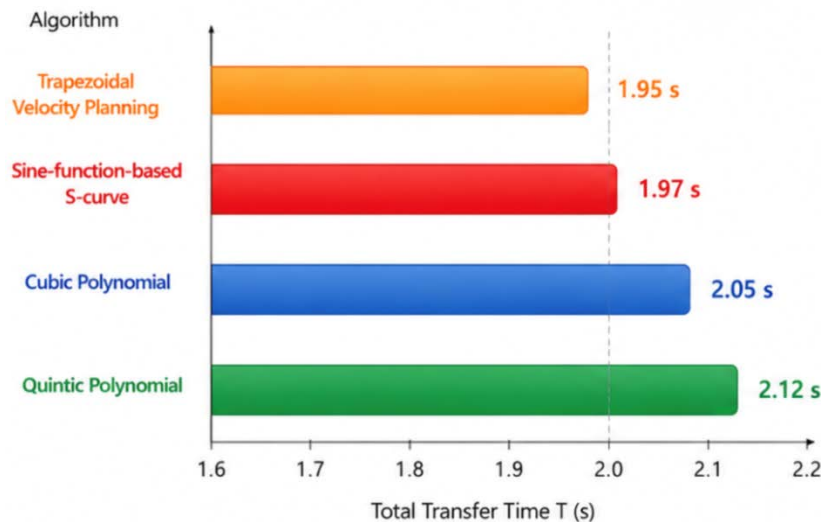


Figure 2: Analysis of Transfer Efficiency

It can be seen that the transfer efficiency of the S-curve trajectory is slightly lower than that of trapezoidal velocity planning, but far higher than that of cubic and quintic polynomials, and it balances smoothness, offering the best comprehensive performance. The specific efficiency time of different algorithms is shown in Figure 2^[6].

4. Conclusion

This paper proposes and demonstrates an S-curve trajectory planning algorithm based on sine functions in response to the motion control requirements of high-speed, high-precision transfer equipment for flexible materials. By constructing a continuously smooth sine acceleration curve, the algorithm fundamentally solves the impact problem caused by abrupt acceleration changes in traditional algorithms. Through citations and simulation verification of the project report content, the following conclusions were reached:

(1)Excellent Smoothness: The algorithm's acceleration curve is continuous without abrupt changes, and the jerk is small, effectively suppressing system vibration, ensuring the stability of flexible materials during the transfer process, and avoiding material detachment or damage.

(2)High Transfer Efficiency: The total time of the algorithm (~1.97s) is significantly better than cubic polynomials (~2.05s) and quintic polynomials (~2.12s), and is close to the efficient trapezoidal planning (~1.95s), meeting the efficiency requirements of automated production.

(3)Superior Comprehensive Performance: This algorithm achieves the optimal balance between smoothness and efficiency and is the ideal solution to the contradiction between impact and efficiency in the high-speed transfer process of flexible materials.

This algorithm has been applied as a core technology in the transfer equipment of this project, providing reliable technical support for the equipment's precise motion control. It effectively resolves problems such as impact and vibration existing in traditional trajectory planning methods, is suitable for automated transfer scenarios of flexible materials, and provides reliable technical support for the motion control of transfer equipment, helping to break through the "last mile" difficulty in smart unmanned factories for flexible material cutting.

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